OPTIMIZING INVENTORY ROUTING PROBLEM WITH DECISION SUPPORT SYSTEM

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Abstract

In this research paper, a decision support system was developed for solving a real inventory routing problem about vending machine products. The system helps to make the decision about everyday products delivery scheduling and provides the decision maker with an opportunity to perform "what if" analysis. The system was tested with a set of real-life instances and the results were compared with the company method.

Keyword: Decision support system, routing problem, replenishment decision

Introduction

Inventory routing problem (IRP) is an important problem in logistics and transportation. In IRP, a central decision maker who manages a fleet of vehicles that makes the deliveries controls the replenishment of inventory for a number of geographically dispersed customers. Therefore, to find an effective vehicle route can help reduce the transportation costs. Despite the practical significance of this class of problems, they are challenging NP-hard problems.

In this research work, we discuss a real problem in close relation to IRP. Below, follows a complete description of this problem. Most soft drinks providers not only supply soft drinks to convenience stores and supermarkets, but also install their own vending machines (VM) in shopping malls, schools, hospitals and even remote areas. The number of vending machines has risen dramatically in the past decade because soft drinks providers are attracted by the benefits of these machines: 24-hour availability, low cost of installation and implementation, and little labour required. This study is inspired by the problem faced by a beverage company that sells canned soft drinks using vending machines covering Lagos, Abuja, Enugu and Port Harcourt. Due to the variation in sales volume, some of the vendor machines require much more frequent replenishment, e.g. three times per week, while others only need to be replenished once a month. Without a systematic forecast and decision analysis process, the replenishment policy and delivery plan are mainly based on the decision-maker's experience. This dependency on experience sometimes makes the refilling process inefficient. Thus, it is important for the company to develop a systematic decision analysis process.

The company's decision maker has developed a replenishment and delivery method which mainly categorizes the vending machines into different groups based on their historical average demand. Six groups of vending machines are clustered under the allocation system: "three visits per week", "two visits per week", "one visit per week", "one visit per two weeks", "one visit per three weeks" and "one visit per month". After clustering the six groups, the decision maker has to plan the vehicle routing every day. The vehicle routing depends on traveling time and vending machine location. Normally, the decision maker chooses the nearest vending machines, or vending machines that can be reached fastest. The characteristics of the current inventory and routing problem for vending machine products are summarized as follows.

(1) The company has almost 50 vending machines covering Lagos, Abuja, and Port Harcourt. Some vending machines can be moved from location to location, but other machines cannot be removed under contract. The company has two different types of vending machines: 16-column and 18-column, with a capacity of 360 and 380 cans respectively.
(2) Vending machines installed in shopping malls, sports complexes and churches usually sell more over the weekend. Those in schools and offices sell more during weekdays. The daily demand for each vending machine is not known until the vehicle visits the machine. However, details of the total number of transactions within the interval between the last and current refill are available.
(3) The company owns one vehicle, which has an approximate capacity of 2000 cans. The driver works from Monday to Saturday. He starts working at 9:00 am and finishes at 6:00 pm, taking a one-hour lunch-break. The vehicle is parked and loaded at a warehouse (depot). After loading, the driver can begin the replenishment from the warehouse. For each visit, the quantity of product replenishment is the maximum level of the vending machine.
(4) Sometime, due to the uncertain inventory level at the vending machine, there may be insufficient items for replenishment in the vehicle. The vehicle then has to return to the depot to refill immediately and wait for the following day to refill the machine.
(5) A number of popular electronic payment devices have been installed in vending machines. Customers can use a credit card and debit card to pay for their purchases or by cash. As a result, some vending machines must be visited at least twice per week in order to collect the data, even though replenishment is not necessary required.

Many methods have been used for solving inventory routing problem and its variations. Generally, they can be categorized into exact methods and approximate methods or, heuristics. The first attempt to integrate IRP into a single model may be found in Chen and Liu, 2018) This paper presents a multi-objective optimization model for IRP in supply chain management, considering factors such as cost, service level, and delivery time. The developed non-linear integer-programming model analyzed a single period, one warehouse, multi-retailer IRP with uncertain demand. Due to the cost considerations and commodity availability, not every customer will be selected to be visited every day. An interchange heuristic was developed to handle the complexity of the IRP. According to (Dong and Liu 2017) study presents a heuristic approach to solve the joint inventory routing and scheduling problem in perishable food supply chains, focusing on minimizing costs and ensuring product freshness. It compared algorithms for the IRP defined over a short planning period. In order to reduce the planning horizon from an annual to a weekly base, the customer set was divided in two subsets. One includes customers whose inventory must be replenished during the given planning period, and the other includes customers whose inventory is only replenished if there is a cost-savings opportunity in doing so. A set of real data from a propane distribution firm in Pennsylvania was used. Dong *et al*, (2017) further developed a heuristic technique to reduce the long-run average problem to a single-period problem. The heuristic consists of an LP-based generalized assignment algorithm modified by the Li & Zhang (2019) algorithm, and an interchange procedure for local improvement. Chen *et al*. (2018) formulated the IRP as a mixed-integer program and the Lagrangean dual ascent method was used to solve several small instances. They focused on a single day approach without treating each day as a completely separate entity. The information on each day is passed to the rest days and used to simulate multiple-day system. The heuristic approach is to select the customers to supply the commodity so that the total profit can be maximized on a single day. Unselected customers with low inventory levels or possibly shortages have penalties imposed. Moreover, unsatisfied demand today will reflect higher revenues tomorrow. Using today's information, penalties and revised revenue the next-day selection can be made as today's procedure. Zhang and Li (2018) addressed the issue in SIRP when the actual demand on a route exceeds the capacity of a vehicle. The pre-determined route cannot be completed and hence the vehicle has to return to the depot to be refilled. Uncompleted route occurrence is referred to as route failure. Four SIRP stages were defined: selection of the customers whose inventory must be replenished during the planning period (stage 1), the assignment of customer to days of planning period (stage 2), the assignment of vehicles to visit a customer on each day (stage 3), and improvements based on stage 3 (stage 4).

The number of commodities delivered while incurring the stock outs and route failures measured the overall performance. The Markov decision process is one of the emerging methods used to model SIRP. It is not surprising to see that researchers and practitioners consider IRP with innovative policies and strategies. Researchers have been actively exploring and developing new methods to solve the Inventory Routing Problem (IRP) more efficiently and effectively. The work done by Liu and Xu (2021) proposed a multi-objective optimization model for the joint inventory routing and scheduling problem in perishable food supply chains, focusing on minimizing costs and ensuring product freshness while maintaining service levels. Wang and Liu (2020) developed a hybrid particle swarm optimization algorithm to solve the multi-objective inventory routing problem, considering factors like cost, service level, and delivery time while maintaining computational efficiency. Zhang and Li (2020) introduced a two-stage stochastic programming model for the multi-echelon inventory routing problem with uncertain demands under the COVID-19 pandemic, considering factors such as inventory costs, transportation costs, and service levels. Zhang and Zhang (2020) presented a fuzzy multi-objective optimization model for the inventory routing problem in uncertain environments, considering factors like cost, service level, and delivery time. Chen and Liu (2021) introduced a novel fuzzy multi-objective optimization model for the inventory routing problem in supply chain management, considering factors like cost, service level, and delivery time under uncertain conditions. These recent citations demonstrate the ongoing efforts to develop innovative methods and models to tackle the Inventory Routing Problem in various contexts and under different uncertainties, ultimately enhancing the efficiency and effectiveness of supply chain management across various sectors.

The main objective of this study is to develop a simple heuristic approach to solve the real inventory routing problem for vending machine products, under which the replenishment policy and delivery plan for the following week's demand can be obtained in advance instead of organizing the trips at time of actual delivery. Moreover, we introduce a decision support system to help the soft drink company make daily routing schedule and provide information to do good business decision. Using computer simulation, we compare performance of the proposed method and the company current method with different performance measures. In addition, with the proposed solution procedure, the decision maker can easily identify the impact of any changes in demand, change in vendor machine location and changes in the number of electronic payment devices installed.

The organization of this paper is as follows. After this introductory section, a frame work of the decision system will be described and a heuristic method for inventory and distribution planning in vendor machine operations is presented in Section 2. A set of data from a Lagos beverage Company is used to test the effectiveness and efficiency of the proposed method and in order to deal with future changes and requirements, sensitivity analysis is then conducted in Section 3. Finally, our conclusions and recommendations are given in Section 4.

Decision Support System

In this study, we develop a special decision support system for the vending machine products delivery. The DSS contains user interface, delivery optimization model based on a heuristic approach to select the vending machine to be served each day and mixed integer linear programming (MILP) to decide the sequence of vendors to be visited, database about everyday delivery and vending machine location information. The DSS can help the decision maker not only make everyday delivery scheduling but also analyze the allocation decision of vending machine for more profit. The architecture of this DSS is shown as Figure 1.

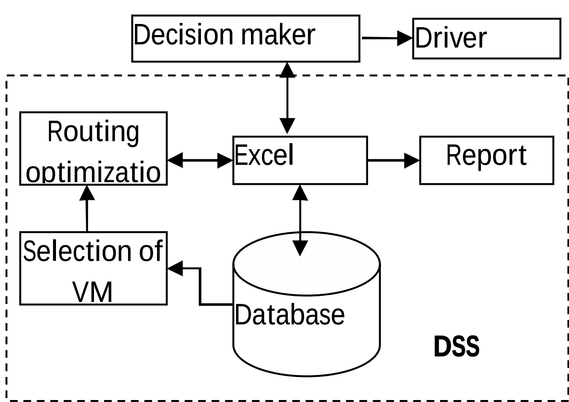

Figure 1. Architecture of decision support system

Trudeau and Dror (1992) elucidated that the Inventory Routing Problem (IRP) encompasses a temporal component – the time of replenishment at the customer's location – and a spatial component – the routing of vehicles traveled. These two components are intricately linked, as the routing decision can impact the timing of replenishment, and conversely, the timing of replenishment directly affects the inventory level, which in turn influences the vehicle routing. Recent research continues to explore and develop innovative methods to address this complex interplay between temporal and spatial aspects of the IRP, ultimately enhancing the efficiency and effectiveness of supply chain management across various sectors. Heuristic algorithms are widely used to handle the complex inventory and distribution problem with uncertain demand (Smith and Graves 2020) Gutin and Bodin (2014) identified that most routing heuristics belong to the two-phase method: the cluster first-route second method and the route first-cluster second method. In the cluster first-route second heuristic method, customers are clustered into groups and then efficient routes are designed for each cluster. In the route first-cluster second heuristic method, a travelling tour is formed among customers and then the tour is divided into different clusters. However, Zhang, Liu and Wang (2019) stated that no heuristics in the route-first cluster second heuristics algorithm can be asymptotically optimal for the stochastic routing problem. In this paper, a heuristic approach, cluster-first route-second, is employed. Figure 2 shows the Flow chart of cluster-first route-second heuristic search method

The proposed heuristic algorithm allows us to reduce the long-run average problem to a single period problem. In the first phase, we determine when and how much to deliver to each customer on each day of the planning period. Then we can identify a set of customers to be served by a single vehicle each day. Zhang et al. (2019) stated that the cost of serving a cluster does not only depend on the geographic locations of the customers in the cluster, but also on whether the customers in the cluster have compatible inventory capacities and usage rates. The selection of vending machines in the cluster formation is based on the penalty imposed on each vending machine. Vendors with the highest penalty are selected for the first routing section. Vending machines are given penalties on two occasions. As suggested by Chen *et al*. (2018) a penalty will be imposed for not visiting vending machines which currently have low inventory levels and which face possible shortages during the day. There are four penalties for different sales levels as shown in Table 1.

Table 1. Penalty levels on sales.

Penalty	Sales
1	10 – 15 %
2	20 – 25 %
3	30 – 45 %
4	>50 %

The secondly penalty is imposed on vendors using the electronic payment systems e.g. Credit and Debit cards. The credit and debit card providers requires each transaction to be transferred to its headquarters within few minutes, otherwise transactions will be voided. The penalty is made when a certain minute has passed after the last replenishment. There are five penalty levels and these are listed in Table 2.

Furthermore, there are two constraints on the selection of vending machines in the clustering process. One is the time constraint: total replenishing time must not exceed total working hours. The other is the capacity constraint: the total number of soft drinks replenished must not exceed the machine's capacity. Vending machines must pass these two constraints to form a cluster.

Table 2. Penalty levels on electronic payment system.

Penalty level	Number of days
1	2
2	3
3	4
4	5
5	6

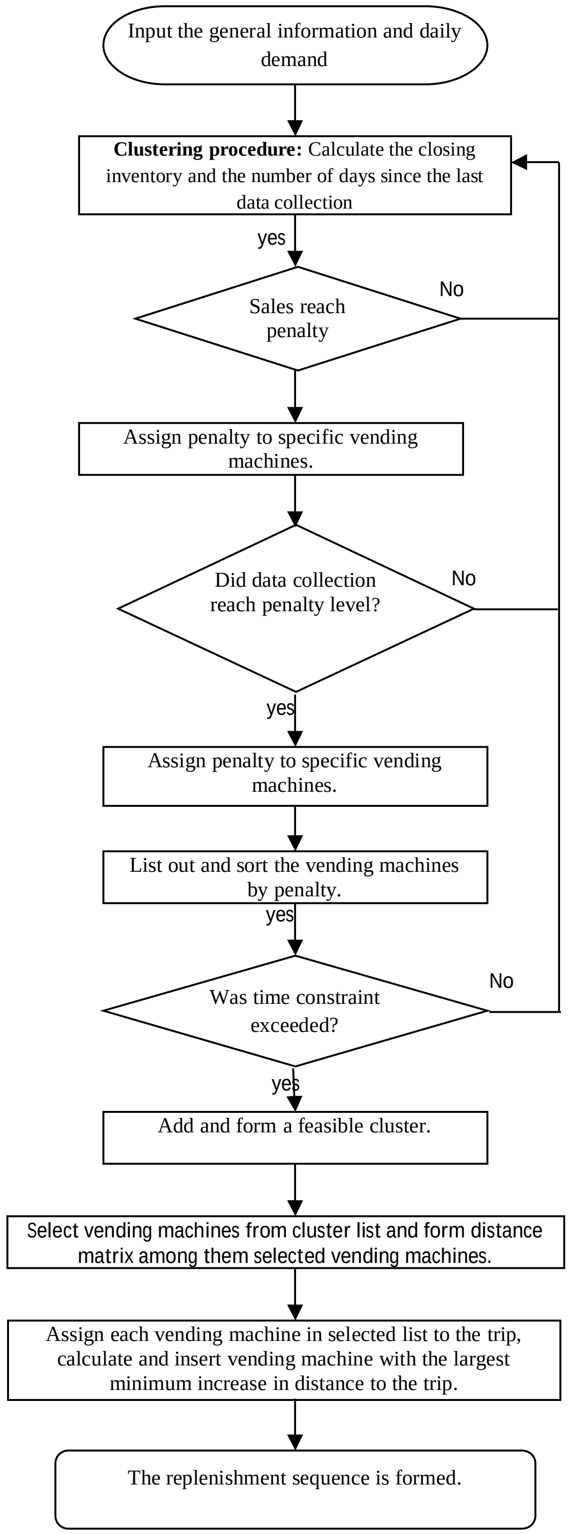

Figure 2: Flow chart of cluster-first route-second heuristic search method

In the second phase, given that we cluster all customers and know how much to deliver to each customer on each day of the planning period, we determine delivery routes visiting customers in the same cluster for each day. Bektas and Laporte (2006) introduced a metaheuristic approach for solving the vendor visit sequence optimization problem, which involves using heuristic algorithms to find near-optimal solutions in large-scale instances. Their method has shown promising results in reducing computation time while still achieving high-quality solutions. Routes with the highest savings are combined into a new feasible route in order to minimize the total distance traveled by the trucks. We propose that the farthest insertion is adopted in order to avoid serious traffic congestion occurring along main roads in the morning, since large numbers of workers move between their homes and their workplace, causing serious congestion in the inner city. If the delivery schedule starts from the farthest vendor, rush hour traffic congestion can be avoided and hence some travel time saved. Figure 3 shows a screen shot of the DSS based on this two-phase heuristic approach.

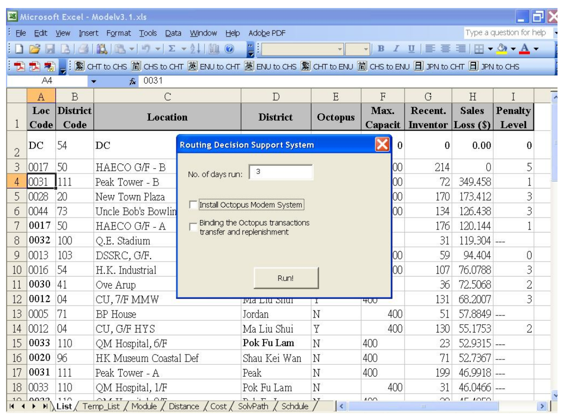

Figure 3. Sample screen of DSS

Numerical Analysis

Performance Measures

In this study, we have identified three major criteria for comparing current replenishment and delivery methods with the proposed method, based on our interviews with key operational personnel in the company and our review of the literature.

(1) Average number of visits: In order to increase the operating efficiency as studied by Trudeau and Dror (1992), the average number of vending machines that can be visited per day should be maximized.

(2) Vending machines with stock out (in %): A study in Computers & Industrial Engineering (2021) presents a dynamic network flow model for sustainable supply chain coordination. Shortages may result in the loss of goodwill as well as revenue. It is important to study the proportion of vending machines with stock outs.

(3) Total transportation cost: One of the major objectives of the vehicle routing problem is to minimize total transportation costs.

Numerical Results

The computer simulation was conducted on a set of daily replenishment schedules for Two years and four months. The two-year period was a warm-up period, which was established afterwards for the purpose of statistical stability. The four-month period (86 days) was used for analysis. Before running the simulation, some assumptions are made:

(1) Deliveries are carried out by the company's own vehicles, and no out-sourcing is allowed.
(2) The warehouse provides an unlimited supply of soft drinks.
(3) The vehicle starts replenishing with a full load of soft drinks.
(4) The vehicle returns to its starting point every day after delivery.
(5) If there is more than one vending machine in the same location, this location is still considered as having one machine.

Average number of visits

Results in Table 3 show that the number of vending machines to be visited improves under the proposed method. The average number of vending machines to be visited has increased to 15, compared with 8.31 vendors under the company method. On average, two more vending machines can be visited each day if the proposed method is adopted. Furthermore, the number of vending machines to be visited per day is more stable under the proposed method, as this has a standard deviation of zero (Figure 4). Under the proposed method, the driver is able to allocate evenly the number of vending machines to be visited. This compares with the current situation where the number of vending machines fluctuates dramatically with a standard deviation of 3.74.

Table 3. Summary of number of visits under original method and proposed method

	Total days	Mean [1]	S.D. [2]	Most [3]	Least [4]
Original method	86	8.31	3.74	14	6
Proposed method	86	15	0	12	12

[1] Average number of vending machines to be visited
[2] Standard deviation of number of vending machines to be visited
[3] The greatest number of vending machines visited in one day
[4] The smallest number of vending machines visited in one day

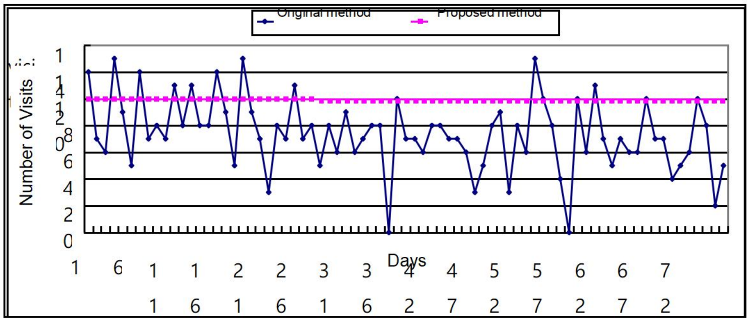

Figure 4. Comparison of original method and proposed method by number of vendors visited

Vending machines with stock outs

It is shown that using proposed method the number of vending machines with stock outs has decreased, dropping by about 11% (as shown in Table 4). Less vending machines experience stock outs when the proposed method is adopted. On average, 0.55 vending machines have stock outs each day under the proposed method, compared with 1.43 vendors under the original method. The quality of replenishment scheduling is therefore significantly improved if the proposed method is adopted.

Table 4. Summary of inventory under original method and proposed method

	Total visited [1]	Without stock outs [2]	Stock outs [3]	Average stock outs [4]
Company method	255	229 (66%)	126 (34%)	1.43
Proposed method	273	216 (78%)	57 (23%)	0.55

[1] Total number of vending machines visited
[2] Number of vending machines without stock outs
[3] Number of vending machines with stock outs
[4] Average number of vending machines with stock outs per day

Total transportation cost

Using the delivery schedule given by the soft drinks provider, the farthest insertion heuristic is used. Results in Table 5 show that using farthest insertion the total distance and total cost are reduced by 6% when compared with the original method. Therefore, using the farthest insertion, replenishment can be effected while traveling a shorter distance and incurring a lower cost.

Table 5. Summary of transportation cost under original method and proposed method

	Total distance (km)	Cost	Distance saved	Cost saved
Original	517.38	₦ 284.230	34.84 km (6%)	₦60 (6%)
Farthest insertion	482.54	₦224.123		

Sensitivity Analysis

This research not only provides a solution to current inventory and distribution challenges but also offers insights for potential management adjustments to adapt to future rapid changes in the three performance measures discussed in the previous section. The study considers the following scenarios:

Demand fluctuations: Assuming a change in demand ranging from a 20% decrease to a 45% increase from the current levels.

Electronic payment systems: Testing the impact of installing electronic payment systems, with the number of devices varying from a 20% reduction to a 20% increase compared to the current installation.

New IT-based transaction method: Assessing the feasibility of a novel IT-based transaction approach.

To evaluate the effects of these changes, a sensitivity analysis is conducted, and the results are presented in the subsequent section.

Demand level

Given the intense competition among vending machines, demand fluctuations can occur frequently over time. These changes in demand often necessitate adjustments to the replenishment schedule to maintain optimal performance. To assess the demand level's impact on vending machines, a test is conducted by varying the demand for each machine. The primary objective of this test is to identify the critical demand threshold, beyond which the distribution system's performance starts to deteriorate. The findings of this test are presented in Table 6.

Table 6. Results of the proposed method with different demand levels

Proposed method / Demand Level	0.55	1.43	1	1.05	1.15	1.55
Vendors with stock outs	5%	7%	10%	14%	17%	24%
Vendors without stock outs	85%	87%	77%	74%	71%	64%

Note: (0.55) - 25% decrease in current demand; (1.43) - 15% decrease in current demand;
(1) - current demand
(1.15) - 15% increase in current demand; (1.15) - 25% increase in current demand;
(1.55) - 50% increase in current demand

Table 7 reveals that the critical demand level is a 50% increase from the current demand, at which point 32% of vending machines face stock outs. As the demand surpasses this threshold, the distribution system's performance becomes unacceptable due to the inability to meet daily demand through regular visits.

To address this issue, if the demand level rises by 50% and the working hours per day are extended by one hour (from 9 hours to 10 hours), approximately two additional vending machines can be serviced. According to Table 7, this adjustment reduces the number of vending machines with stock outs to 25%. Consequently, if the demand level reaches 100% of the current demand, the soft drink provider should consider extending working hours to effectively manage the significant stock out problem.

Table 7. Results of the proposed method with different working hours

Demand	100%	100%
Working hours (per day)	8	9
Machines with stock outs	32%	25%
Machines without stock outs	68%	75%

Electronic payment system

An electronic payment system for vending machines refers to a technology-driven solution that enables customers to pay for their purchases using various electronic means, rather than traditional cash transactions. A test on the number of electronic payment system is carried out and the results are summarized in Table 8. These show that when the total number of electronic payment system installed increases, the percentage of vendors with stock outs increases. This modern approach to vending machine payments offers several advantages, such as increased convenience, improved security, and potential for enhanced data analysis.

By implementing an electronic payment system, vending machine operators can cater to a wider range of customers, including those who prefer not to carry cash or those who have transitioned to a cashless lifestyle. This can lead to increased sales and customer satisfaction.

Additionally, electronic payment systems often incorporate security features that reduce the risk of theft and vandalism, which can lower operational costs for vending machine owners.

Table 8. Results of proposed method with different number of electronic payment system

Proposed method	0.55	1.43	1	1.05	1.15
Machines with stock outs	15%	14%	14%	14%	18%
Machines without stock outs	85%	86%	86%	86%	82%

Note: (0.55) - 25% decrease in total number of payment system installed; (1.43) - 15% decrease in total number of payment system installed; (1) - current number of payment systems installed; (1.05) - 15% increase in total number of payment system installed; (1.25) - 25% increase in total number of payment system installed

Conclusion

This research paper demonstrates the superiority of a decision support system over the current manual routing system employed by decision-makers. The proposed solution offers several advantages when applied to the same demand figures, including:

- An increase of 37% in the average number of vendors that can be visited.
- A reduction of 11% in the number of vending machines experiencing stock outs.
- A 6% decrease in transportation costs.

While the developed model provides a systematic and efficient inventory management policy for vending machine products, it does have some limitations. These include:

- The potential inaccuracy of distance measurements between vending machines, which may affect the schedule's validity. To strengthen this aspect, more data and further studies are necessary.
- The impact of seasonal factors and intense competition in the soft drinks industry, causing significant fluctuations in daily demand

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